Homework 5 Solutions

Ch. 8 2,4,5,10,18,23,24

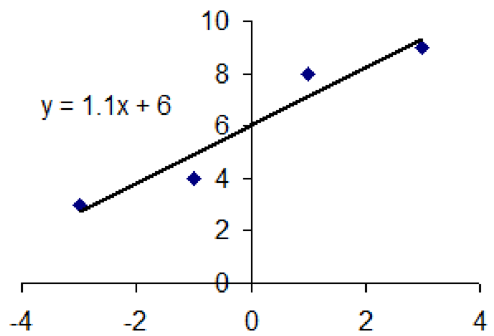
8.2

$$A = \frac{\sum x^2 \sum y - \sum x \sum xy}{N\sum x^2 - \left(\sum x\right)^2} = \frac{20 \cdot 24 - 0 \cdot 22}{4 \cdot 20 - 0^2} = 6$$

$$B = \frac{N\sum xy - \sum x \sum y}{N\sum x^2 - \left(\sum x\right)^2} = \frac{4 \cdot 22 - 0 \cdot 24}{80} = 1.1$$

Therefore, the equation for the line is

$$y = 1.1x + 6.0$$



8.4

From equation 8.8,

$$AN + B\sum x = \sum y$$

Divide by *N*:

$$A + B\frac{\sum x}{N} = \frac{\sum y}{N} \qquad \text{or} \qquad A + B\bar{x} = \bar{y}$$

Therefore, the point $(\bar{x}, \bar{y})$ lies on the best-fit line.

8.5

For a line through the origin, $A = 0$, so $\chi^2 = \sum \frac{(y_i - Bx_i)^2}{\sigma_y^2}$

Minimize this with respect to B: $\frac{\partial \chi^2}{\partial B} = \frac{-2}{\sigma_y^2}\sum x_i (y_i - Bx_i) = 0$

Therefore, $\sum x_i y_i - B\sum x_i^2 = 0$ or $B = \frac{\sum xy}{\sum x^2}$

8.10

For the weighted fit, the weights are inversely proportional to the uncertainies squared:

$$w_i = \left(1/0.5^2, 1/0.5^2, 1/1^2\right) = (4,4,1)$$

Using these weights, calculate A and B:

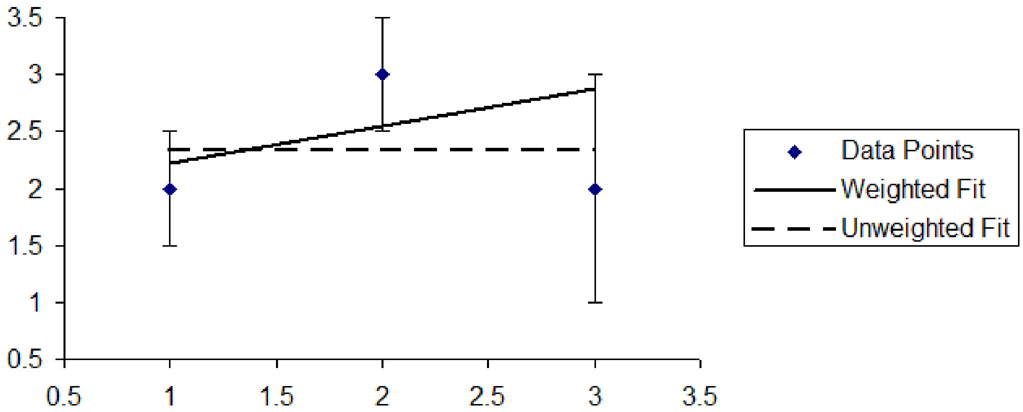
$$A = \frac{\sum wx^2 \sum wy - \sum wx \sum wxy}{\sum w \sum wx^2 - \left(\sum wx\right)^2} = \frac{29 \cdot 22 - 15 \cdot 38}{9 \cdot 29 - 15^2} = 1.89$$

$$B = \frac{\sum w \sum wxy - \sum wx \sum wy}{\sum w \sum wx^2 - \left(\sum wx\right)^2} = \frac{9 \cdot 38 - 15 \cdot 22}{36} = 0.33$$

Now calculate A and B for unweighted data points:

$$A = \frac{\sum x^2 \sum y - \sum x \sum xy}{N \sum x^2 - \left(\sum x\right)^2} = \frac{14 \cdot 7 - 6 \cdot 14}{3 \cdot 14 - 6^2} = 2.33$$

$$B = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - \left(\sum x\right)^2} = \frac{3 \cdot 14 - 6 \cdot 7}{6} = 0.00$$



8.18

We have $B = \frac{\sum xy}{\Delta}$, where Δ does not depend on y. To calculate the uncertainty in B, we need its derivatives with respect to the y's:

$$\frac{\partial B}{\partial y_i} = \frac{x_i}{\Delta}$$

Therefore, $\sigma_B^2 = \frac{\sum \left(x_i \sigma_y\right)^2}{\Delta^2} = \sigma_y^2 \frac{\sum x^2}{\left(\sum x^2\right)^2} = \frac{\sigma_y^2}{\sum x^2}$ $\qquad \sigma_B = \frac{\sigma_y}{\sqrt{\sum x^2}}$

8.23

We have $\chi^2 = \sum (Af_i + Bg_i - y_i)^2 / \sigma_y^2$, where $f_i \equiv f(x_i)$ and $g_i \equiv g(x_i)$

Minimize this with respect to A and B:

$$\frac{\partial \chi^2}{\partial A} \propto \sum f_i (Af_i + Bg_i - y_i) = 0 \qquad \frac{\partial \chi^2}{\partial B} \propto \sum g_i (Af_i + Bg_i - y_i) = 0$$

Simplify this a bit:

$$A\sum f^2 + B\sum fg = \sum fy \qquad A\sum fg + B\sum g^2 = \sum gy$$

These are the equations given.

8.24

We will use the equations from the problem above. First, calculate the function at the various values of x and use this to calculate the coefficients in the equations. The results are:

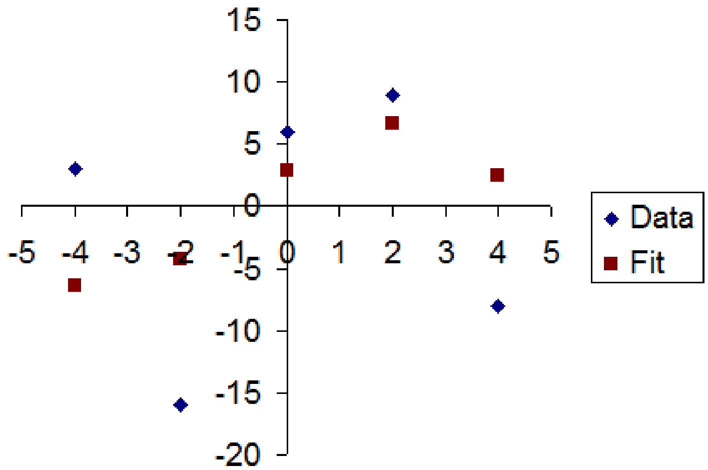
$$\sum f^2 = 2.22 \quad \sum g^2 = 2.78 \quad \sum fg = 0.00 \quad \sum fy = 6.48 \quad \sum gy = 14.63$$

The equations become

$$2.22A = 6.48 \qquad 2.78B = 14.63$$

Therefore,

$$A = 2.92 \qquad B = 5.26$$



The fit is rather poor – it is likely the frequency is not correct.